Modular Arithmetic

Addition

Addition in modular arithmetic is sometimes called Clock Arithmetic. Addition done not on a number line, but on a circle.

Mod 3

Mod three means that there are 3 numbers in the base set to compute with: 0, 1, 2
After you get to 2 (the last number), adding 1 means you start around again.

Symbol and Meanings chart

Symbol	Meaning
≡	Equivalence (i.e. 2 numbers which differ by a multiple of the modulus)
=	Equality (as used in standard mathematics)

Example: 2 + 2 ≡ 1 (mod 3)
 And 2 + 1 ≡ 0   The numbers 2 & 1 are **"additive inverses"**.
Note: Additive Inverses occur when two numbers are added together and their sum is 0

Mod 3 Addition	0	1	2
0			
1			
2			

Mod 5

Mod 5 means that there are 5 numbers in the base set: 0, 1, 2, 3, 4
After you get to 4, adding 1 means you start around again.

Example: 3 + 4 ≡ 2 mod 5   The additive inverses (mod 5) are:
                        1 & 4 and 2 & 3;

Mod 5 addition	0	1	2	3	4
0					
1					
2					
3					
4					

Mod 7

Mod seven means that there are _____ numbers in the base set: ___, ___, ___, ____,

___, ____, _____

After you get to ___ , adding 1 means _____.

Examples:

1. 3 + 4 ≡ _____ (mod 7);   2. 5 + 5 ≡ _____ (mod 7);

3. 2 + 6 ≡ _____ (mod 7);   4. 1 + 5 ≡ _____(mod 7);

The additive inverses of (mod 7) are: 1 & 6; 2 & ___; 3 & ____; 4 & ____; 5 & ____

Mod 7 Addition						

Subtraction in Modular Arithmetic

Subtraction means add the negative of the number in a counter-clockwise direction.

Mod 5

  3 + (-1) ≡ 2 (mod 5);   1 + (- 4) ≡ 2 (mod 5);   3 + (-4) ≡ 4 (mod 5)

There are at least two ways for you to think about modulus subtraction. Clock arithmetic and counter- clockwise counting is one way and using additive inverses is another.

Problem	Additive Inverse Method	reason
3 + (- 1) ≡ 2 (mod 5)	3 + 4 ≡ 7 ≡ 2 (mod 5)	4 & 1 are additive inverses
1 + (- 4) ≡ 2 (mod 5)	1 + 1 ≡ 2 (mod 5)	1 & 4 are additive inverses
3 + (- 4) ≡ 4 (mod 5)	3 + 1 ≡ 4 (mod 5)	1 & 4 are additive inverses

Examples:

      5. $1 - 3 \equiv$ _____(mod 5)   6. $3 - 4 \equiv$ _____(mod 5)

      7. $4 - 3 \equiv$ _____(mod 5)   8. $0 - 2 \equiv$ _____(mod 5)

Subtraction Table (mod 5)

Mod 5 Row - Column	0	1	2	3	4
0					
1					
2					
3					
4					

Mod 7

      $5 - 5 = 5 + (-5) = 0$ (mod 7)   OR $5 + 2 \equiv 0$ (mod 7)

      $3 - 6 = -3 \equiv 7 - 3 \equiv 4$ (mod 7)   OR $3 + 1 \equiv 4$ (mod 7)

Subtraction Table (Mod 7)

Mod 7 Row-Column							

Practice Problems

Mod 9

9. $7 - 6 \equiv$ _____

10. $3 - 8 \equiv$ _____

11. $6 - 7 \equiv$ _____

12. $8 - 5 \equiv$ _____

Mod 10

13. $7 - 6 \equiv$ _____

14. $3 - 8 \equiv$ _____

15. $6 - 7 \equiv$ _____

16. $8 - 5 \equiv$ _____

Mod 11

17. $7 - 6 \equiv$ _____

18. $3 - 8 \equiv$ _____

19. $6 - 7 \equiv$ _____

20. $8 - 5 \equiv$ _____

21. $6 - 10 \equiv$ _____

22. $1 - 5 \equiv$ _____

Mod 12

23. $7 - 6 \equiv$ _____

24. $3 - 8 \equiv$ _____

25. $6 - 7 \equiv$ _____

26. $8 - 5 \equiv$ _____

27. $6 - 10 \equiv$ _____

28. $1 - 5 \equiv$ _____